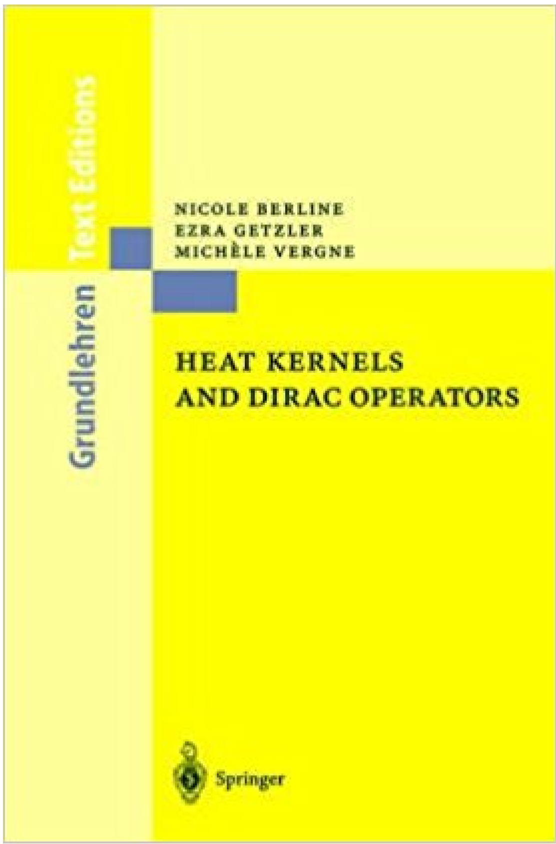
Heat Kernels And Dirac Operators (Grundlehren Der Mathematischen Wissenschaften)



Synopsis

The first edition of this book presented simple proofs of the Atiyah-Singer Index Theorem for Dirac operators on compact Riemannian manifolds and its generalizations (due to the authors and J.-M. Bismut), using an explicit geometric construction of the heat kernel of a generalized Dirac operator; the new edition makes this popular book available to students and researchers in an attractive softcover. The first four chapters could be used as the text for a graduate course on the applications of linear elliptic operators in differential geometry and the only prerequisites are a familiarity with basic differential geometry. The next four chapters discuss the equivariant index theorem, and include a useful introduction to equivariant differential forms. The last two chapters give a proof, in the spirit of the book, of Bismut's Local Family Index Theorem for Dirac operators.

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Customer Reviews

Aus den Rezensionen: "Ã¢â ¬Â¦ Das vorliegende Buch ist die zweite korrigierte und erweiterte Ausgabe eines Werkes aus dem Jahre 1992. Ã¢â ¬Â¦ Ausgehend von einer Grundausbildung in klassischer Differentialgeometrie stellt das Buch alle zum VerstÃƒÂ¤ndnis des Beweises notwendigen Voraussetzungen zur VerfÃƒÂ gung. Dadurch eignet es sich einerseits zum Selbststudium fÃƒÂ r Studierende mit entsprechender Vorbildung Ã¢â ¬Â¦ andererseits als Grundlage einer Vorlesung ÃƒÂ ber dieses ergiebige Thema." (P. Grabner, in: IMN - Internationale Mathematische Nachrichten, 2006, Issue 202, S. 45) Ã Â --This text refers to an out of print or unavailable edition of this title.

In the first edition of this book, simple proofs of the Atiyah-Singer Index Theorem for Dirac operators on compact Riemannian manifolds and its generalizations (due to the authors and J.-M. Bismut) were presented, using an explicit geometric construction of the heat kernel of a generalized Dirac operator; the new edition makes this popular book available to students and researchers in an attractive paperback. The first four chapters could be used as the text for a graduate course on the applications of linear elliptic operators in differential geometry and the only prerequisites are a familiarity with basic differential geometry. The next four chapters discuss the equivariant index theorem, and include a useful introduction to equivariant differential forms. The last two chapters give a proof, in the spirit of the book, of Bismut's Local Family Index Theorem for Dirac operators. This book will be of interest to graduate students and researchers in differential geometry, Arakelov geometry, group representation theory and mathematical physics. --This text refers to an out of print or unavailable edition of this title.

Dirac operators on Riemannian manifolds are of fundamental importance in differential geometry: they occur in situations such as Hodge theory, gauge theory, and geometric quantization.The book is based on a simple principle: Dirac operators are a quantization of the theory of connections, and the supertrace of the heat kernel of the square of a Dirac operator is the quantization of the Chern character of the corresponding connection. From this point of view, the index theorem for Dirac operators is a statement about the relationship between the heat kernel of the square of a Dirac operator and the Chern character of the associated connection. This relationship holds at the level of differential forms and not just in cohomology, and leads to think of index theory and heat kernels as a quantization of Chern-Weil theory. The importance of the heat kernel is that it interpolates between the identity operator and the projection onto the kernel of the Dirac operator. However, the authors study the heat kernel, and more particularly its restriction to the diagonal, in its own right, and not only as a tool in understanding the kernel of the Dirac operator.The authors attempt to express allof their constructions in such a way that they generalize easily to the equivariant setting, in which a compact Lie group acts on the manifold and leaves the Dirac operator invariant. They consider the most general type of Dirac operators, associated to a Clifford module over a manifold, to avoid restricting to manifolds with spin connections. They also work within Quillen's theory of superconnections.The book is not necessarily meant to be read sequentially, and consists of four groups of chapters: (1) Chapters 1 and 7, the former giving various preliminary results in differential geometry and the latter on equivariant differential forms; they do not depend on any other chapters.

(2) Chapters 2, 3, and 4 introduce the main ideas of the book, and take the reader through the main properties of Dirac operators, culminating in the local index theorem. (3) Chapters 5, 6, and 8 are on the equivariant index theorem, and may be read after the first four chapters, although Chapter 7 is needed in Chapter 8. (4) Chapters 9 and 10 are on the family index theorem, and can be read after the first four chapters, except sections 9.4 and 10.7 which have Chapter 8 as a prerequisite.The book is intended for researchers and advanced graduate students; you need a very strong background in differntial geometry, algebraic topology, harmonic analysis, and hypercomplex analysis to read it. The style is definitely French, so if you have had trouble with Bourbaki be prepared. The list of references is adequately long. Very nice printing and binding quality.

This monograph treats the modern theory of the heat equation on manifolds, with special emphasis on recent work related to the Atiyah-Singer index theorem, and in particular Bismut's contributions. Much of this work has been concerned with finding explicit differential-form representatives of the cohomology classes that arise in the index formula and its generalizations.The authors have completely eliminated the probability theory that figures so notoriously in Bismut's papers, replacing it by the more classical asymptotic expansions. (However, I must say that my study of Bismut's papers goaded me into learning the probability theory, and I'm glad I did; for apart from being interesting in itself, it also proved very useful in my thesis). Moreover, they have also managed to eliminate almost all of the analysis. What remains is a fair amount of differential geometry and a great deal of algebra.In those parts of the book that are written very concisely, readers will have trouble supplying full details. An example is Sec. 1.6 on the Euler and Thom classes; even my thesis adviser did not understand the algebra of differential forms, which becomes confusing given all the various pullback bundles; here understanding the algebra in Lemma 1.51 is the crux of the matter. Another example of extreme conciseness is Sec. 3.6, which sketches the standard Clifford modules in the important cases -- De Rham, signature, spin, Kahler. In general, just enough information is provided to enable a well-motivated graduate student to fill in the details and/or acquire the necessary background. Doing this took me the better part of two years.On the other hand, the parts of the book dealing directly with heat kernels are written less concisely and are therefore more readable. The book gives a nice construction of the heat kernel for generalized Laplacians (Thm. 2.30) in Chap. 2, which is devoted to the asymptotic expansion of the heat kernel, essentially following Hadamard's classical approach; however the treatment is highly algebraic. Chapter 6, based on work of Berline and Vergne, re-covers much of the same ground from the viewpoint of equivariant vector bundles; it has a more overtly differential-geometric flavor. Chapters

9 and 10 on the index bundle and Bismut’s version of the index theorem for families are again quite readable and again highly algebraic.Seeley’s work on pseudodifferential operators, which played such an important role in the original proof of the Atiyah-Singer index theorem, of course has its counterpart here in the asymptotic expansion, but the treatment makes it seems rather innocuous if not quite trivial. In general, the lack of "hard analysis" in the book is striking. Except for a cameo appearance in the short Chap. 7 on equivariant differential forms, Fourier analysis, for example, plays no role.Researchers already active in the field will probably benefit the most from this book, but fun-loving grad students can also profit from it.

The book is devoted to the investigation of Dirac operators , in particular to explaining the relation of the Atiyah-Singer formula for their index with the heat kernel expansion. Dirac operators on Riemannian manifolds were introduced by Lichnerowicz, Atiyah and Singer. Most first order linear differential operators of geometric origin are Dirac operators. The main technique in the book is an explicit construction of the kernel of the heat operator $e\sp{-tD\sp 2}$ associated to the square of the Dirac operator D. Note that in the proofs of many theorems due to J. M. Bismut, the original use of probability theory is replaced by the applications of classical asymptotic expansion methods. List of Chapters: Introduction, 1. Background on Differential Geometry, 2. Asymptotic Expansion of Heat Kernel, 3. Clifford Modules and Dirac Operators, 4. Index Density of Dirac Operators, 5. The Exponential Map and the Index Density, 6. The Equivariant Index Theorem, 7. Equivariant Differential Forms, 8. The Kirillov Formula for the Equivariant Index, 9. The Index Bundle, 10.The Family Index Theorem.

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